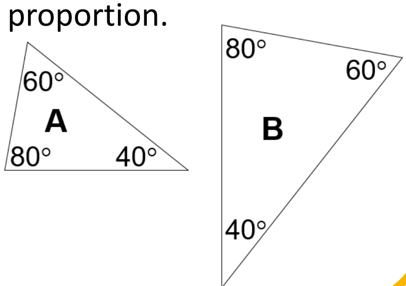
ENLARGEMENT, SIMILARITY & CONGRUENCE

Key Concept

Properties of similar shapes:

- The corresponding angles will be the same if shapes are similar.

- Corresponding edges must remain in proportion.



sparx

U110, U630
M139

Key Words

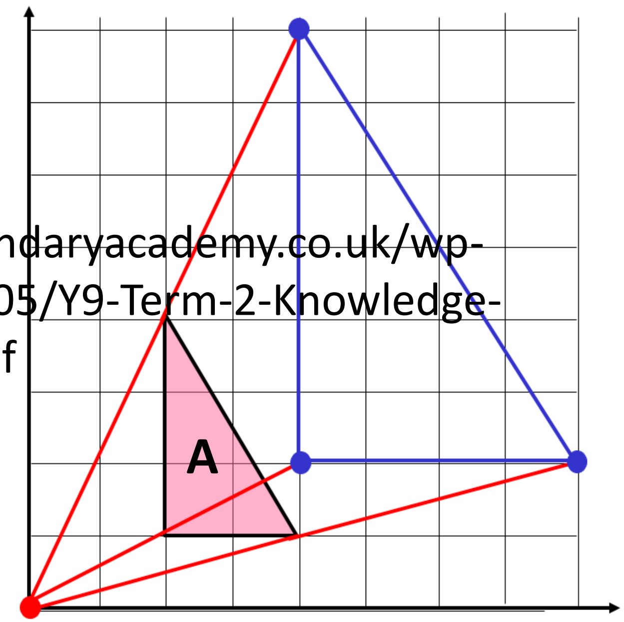
Transformation: This means something about the shape has 'changed'.
Reflection: A shape has been flipped.
Rotation: A shape has been turned.
Translation: A movement of a shape.
Enlargement: A change in size, either bigger or smaller.
Congruent: These shapes are the same shape and same size but can be in any orientation.
Similar: Two shapes are mathematically similar if one is an enlargement of the other.

Tip

To find the centre of enlargement connect the corresponding vertices.

Examples

Enlarge shape A, scale factor 2, centre (0, 0).



Scale factor 2 - Double the distance between each vertex and the centre of enlargement.

Questions

1) A triangle has lengths 3cm, 4cm and 5cm. What will they be if enlarged scale factor 3.
2) Rectangle A measures 3cm by 5cm, B measures 15cm by 25cm. What is the scale factor of enlargement?

ANSWERS: 1) 9cm, 12cm and 15cm 2) 5.

TRANSLATION AND ENLARGEMENT

Key Concepts

A **translation** moves a shape on a coordinate grid. Vectors are used to instruct the movement:

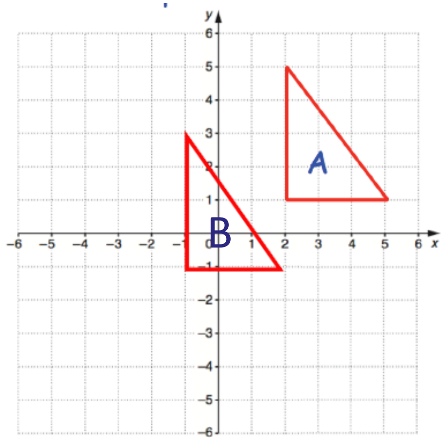
$\begin{pmatrix} x \\ y \end{pmatrix}$

- x: Positive-Right, Negative - Left
- y: Positive-Up, Negative - Down

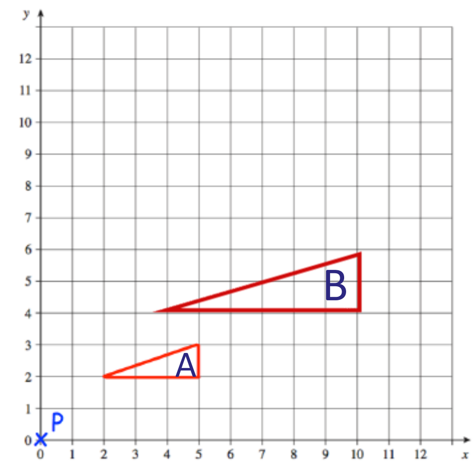
An **enlargement** changes the size of an image using a scale factor from a given point.

Examples

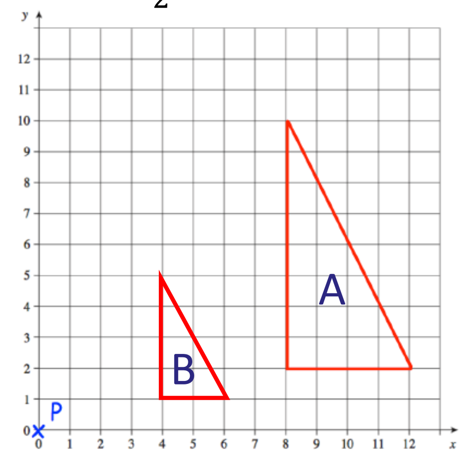
Translate shape A by $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$. Label it B



Enlarge shape A by scale factor 2 from point P.



Enlarge shape A by scale factor $\frac{1}{2}$ from point P.



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U196

U519

U134

Key Words

Translation

Enlargement

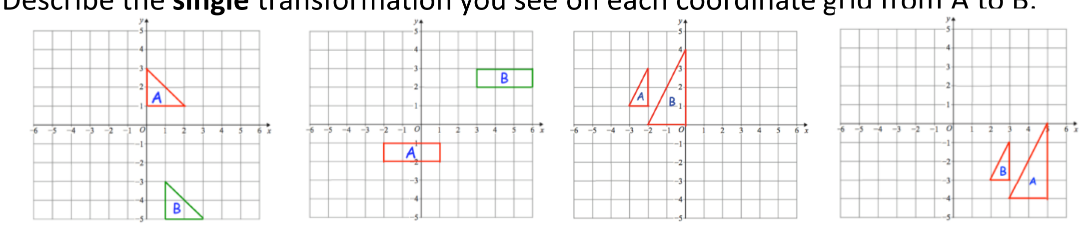
Scale factor

Centre

Positive

Negative

Describe the **single** transformation you see on each coordinate grid from A to B:



ANSWERS: a) translation $\begin{pmatrix} 1 \\ -6 \end{pmatrix}$ b) translation $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ c) enlarge, centre (-4,2) scale factor 2

d) enlarge, centre (1,-2) scale factor $\frac{1}{2}$

DIVIDING AN AMOUNT INTO RATIOS

Key Concepts

An amount can be divided into a given ratio.

Red : Green
1 : 3

For every 1 red there are 3 greens.

A ratio can be converted into fractions.

Red : Green
1 : 3

$\frac{1}{4}$ are red and $\frac{3}{4}$ are green.

Examples

A woman has £400. She is going to split her money between her two children in the ratio 2:3. How much does each child receive?

No. of boxes (2+3)

$400 \div 5$
$= 80$

2 : 3

2	3
$\boxed{80}$	$\boxed{80}$
$\boxed{80}$	$\boxed{80}$
	$\boxed{80}$
£160	£240

Child 1 receives £160 and Child 2 receives £240.

There are boys and girls at a party in the ratio 5:2.
There are 15 more boys than girls.
Calculate the number of people at the party.

No. of extra Boxes (5-2)

$15 \div 3$
$= 5$

5 : 2

5	2
$\boxed{5}$	$\boxed{5}$
$\boxed{5}$	$\boxed{5}$
$\boxed{5}$	
$\boxed{5}$	
$\boxed{5}$	

7×5
$= 35$ people

sparx

M885, M801, M267, M525,

Key Words

Ratio
Divide
Parts

1) Ann made some cakes. She made vanilla cakes and chocolate cakes in the ratio 2:9. What fraction of the cakes were chocolate?
2) Share £25 in the ratio 7:3
3) Katy and Becky share some money in the ratio 2:1. Katy receives £10 more than Becky. How much do they each receive?
4) Claire and John share some money in the ratio 3:2. Claire receives £18. How much does John receive?

ANSWERS 1) $\frac{9}{11}$ 2) £17.50, £7.50 3) £20, £10 4) £12

RATIO AND DIRECT PROPORTION

Key Concepts

To calculate the **value** for a single item we can use the **unitary method**.

When working with best value in monetary terms we use:

$$Price\ per\ unit = \frac{price}{quantity}$$

In recipe terms we use:

$$Weight\ per\ unit = \frac{weight}{quantity}$$

If 20 apples weigh 600g. How much would 28 apples weigh?

600 ÷ 20 = 30g ⟶ weight of 1 apple

30 × 28 = **840g**

Box A has 8 fish fingers costing £1.40.
Box B has 20 fish fingers costing £ 3.40.
Which box is the better value?

$$A = \frac{£1.40}{8} = £0.175 \qquad B = \frac{£3.40}{20} = £0.17$$

Therefore Box B is better value as each fish finger costs less.

Examples

The recipe shows the ingredients needed to make 10 Flapjacks.
How much of each will be needed to make 25 flapjacks?

Ingredients for 10 Flapjacks

80 g rolled oats

60 g butter

30 *ml* golden syrup

36 g light brown sugar

Method 1: Unitary

80 ÷ 10 = 8
8 × 25 = **200g**

30 ÷ 10 = 3
3 × 25 = **75g**

60 ÷ 10 = 6
6 × 25 = **150g**

36 ÷ 10 = 3.6
3.6 × 25 = **90g**

Method 2: 5 flapjacks

80 ÷ 2 = 40
40 × 5 = **200g**

30 ÷ 2 = 15
15 × 5 = **75g**

60 ÷ 2 = 30
30 × 5 = **150g**

36 ÷ 2 = 18
18 × 5 = **90g**

sparx

M525

M801

Key Words

Unitary
Best Value
Proportion
Quantity

Ingredients
to make **16** gingerbread men

180 g	flour
40 g	ginger
110 g	butter
30 g	sugar

1) How much will we need to make 24 gingerbread men?

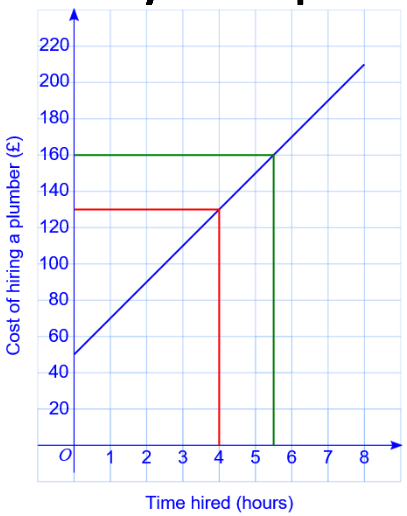
2) Packet A has 10 toilet rolls costing £3.50.
Packet B has 12 toilet rolls costing £3.60.
Which is better value for money?

3) If 15 oranges weigh 300g. What will 25 oranges weigh?

ANSWERS 1) 270g flour, 60g ginger, 165g butter, 45g sugar 2) Packet B 30p per roll 3) 500g

APPLIED GRAPHS

Key Concept

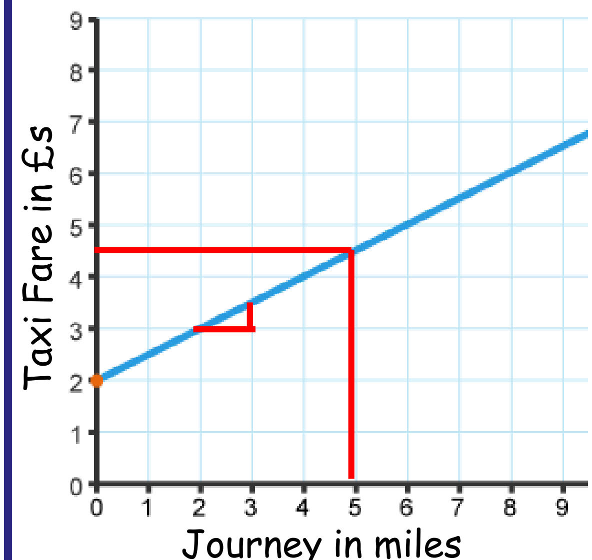


Gradient – The extra cost incurred for every extra hour.
y-intercept – The minimum payment to the plumber.

Key Words

Conversion graph: A graph which converts between two variables.
Intercept: Where two graphs cross.
y-intercept: Where a graph crosses the y-axis.
Gradient: The rate of change of one variable with respect to another. This can be seen by the steepness.
Simultaneous: At the same time.

Examples



What is the minimum taxi fair?
£2, this is the y-intercept.

What is the charge per mile?
50p, every extra mile adds on 50p.

How much would a journey of 5 miles cost?
£4.50, See line drawn up from 5 miles to the graph, then drawn across to find the cost.

sparx

M932, M658
M843, M771

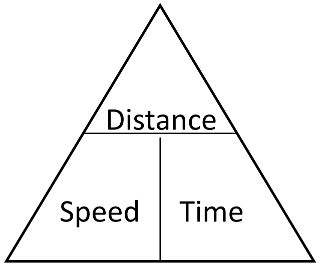
Tip
The solution to two linear equations with two unknowns is the coordinates of the intercept (where they cross).

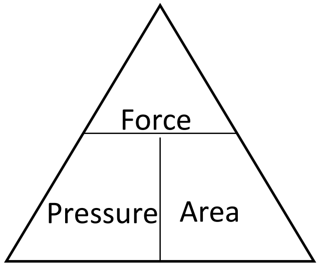
Questions
1) For the graph above a) A journey is 8 miles, what is its cost?
b) A journey cost just £3, how far was the journey?
2) Draw a graph to show the exchange rate £1 = $1.4.

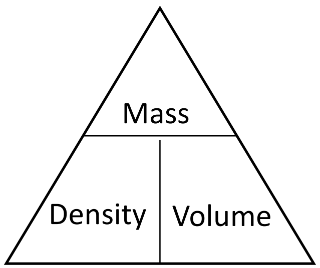
ANSWERS: 1) a) £6 b) 2 miles

COMPOUND MEASURES

Key Concepts





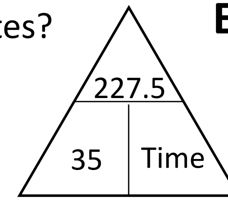


Examples

A car is travelling at a speed of 35mph and is scheduled to travel 227.5 miles. How long will this take in hours and minutes?

$$Time = \frac{distance}{speed}$$

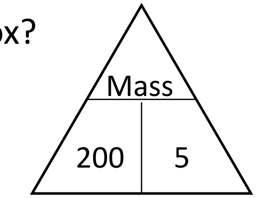
$$Time = \frac{227.5}{35} = 6.5 hours = 6\ hours\ 30\ minutes$$



A 5m³ box has a density of 200g/m³. What is the mass of the box?

$$Mass = Density\ \times Volume$$

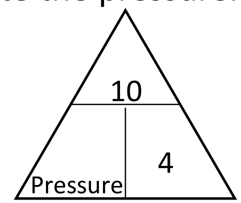
$$Mass = 200 \times 5 = 1000g$$



10N of force are applied to a block with area 4m². Calculate the pressure.

$$Pressure = \frac{force}{area}$$

$$Pressure = \frac{10}{4} = 2.5N/m^2$$



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U151 U910 U527

Key Words

Speed
Distance
Time
Pressure
Force
Area
Density
Mass
Volume

1) A block exerts a force of 120 Newtons on the ground. The block has an area of 2 m².
Work out the pressure on the ground.

2) A piece of gold has a mass of 760 grams and a volume of 40 cm³.
Work out the density of the piece of gold.

3) Dani leaves her house at 08 00. She drives 63 miles to work. She drives at an average speed of 27 miles per hour. At what time does Dani arrive at work?

ANSWERS 1) 60N/m² 2) 19g/cm³ 3) 10:20am

CONVERSION OF METRIC UNITS

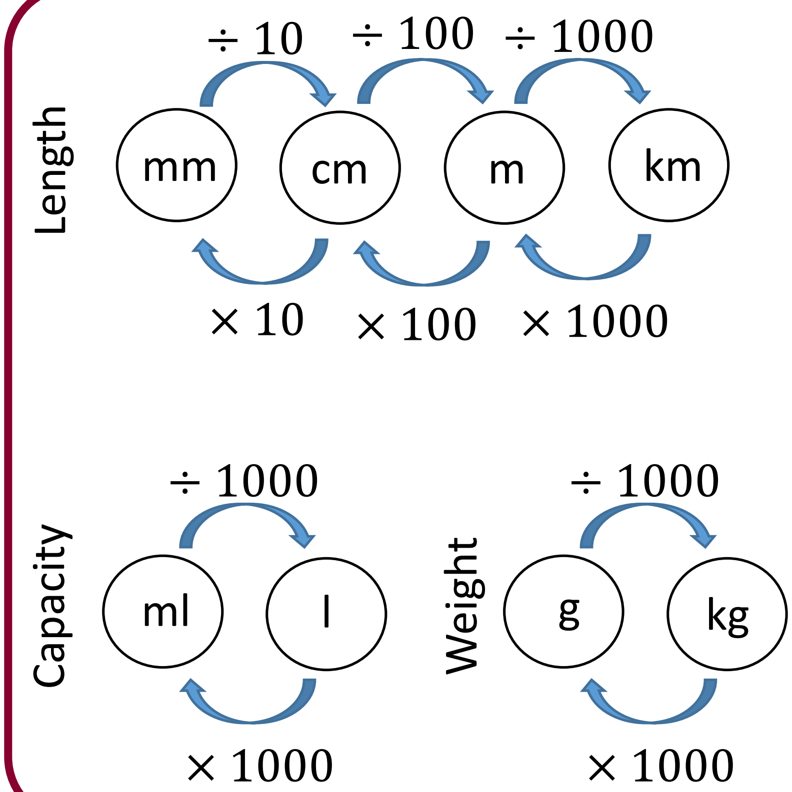
Key Concept

Metric units of **length**:
mm, cm, m, km

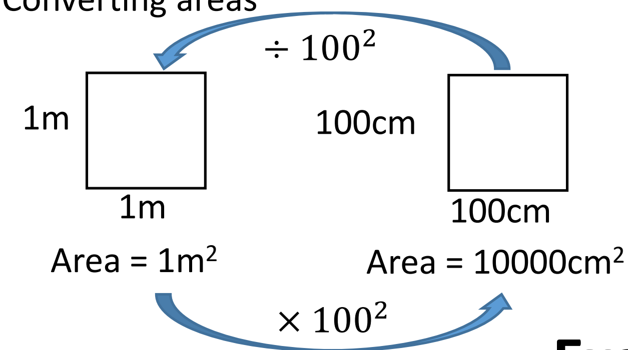
Metric units of **weight**:
g, kg

Metric units of **capacity**:
ml, l

All of these units are **metric** units. They will always use conversions of multiples of 10, eg.10, 100, 1000 etc.

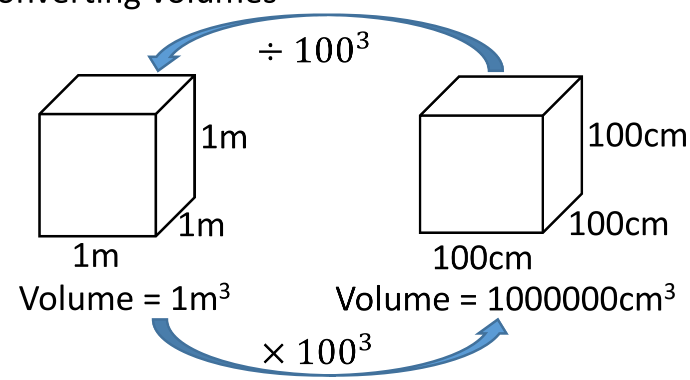


Converting areas



Examples

Converting volumes



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M487

Key Words
Length
Weight
Capacity
Metric

Convert each of the following:

a) 12cm into mm
b) 1783g into kg
c) 2.5 litres into ml
d) 6.8m into mm
e) $5000000cm^3$ into m^3
f) $2m^2$ into cm^2

ANSWERS: a) 120mm b) 1.783kg c) 2500ml d) 6800mm e) $5m^3$ f) $20000cm^2$

KINEMATIC FORMULAE AND CONVERSION OF UNITS

Key Concepts

a *is constant acceleration*
u *is initial velocity*
v *is final velocity*
s *is displacement from the position when the time* $= 0$

$$v = u + at$$

Velocity is speed in a given direction.

$$s = ut + \frac{1}{2}at^2$$

Initial velocity is speed in a given direction at the start of the motion.

$$v^2 = u^2 + 2as$$

Acceleration is the rate of change of velocity i.e. how the speed changes with time

Examples

Write $90km/h$ in m/s.

$90km/h$
$\downarrow \times 1000$
$90000m/h$
$\downarrow \div 60$
$1500m/min$
$\downarrow \div 60$
$25m/sec$

Write $72mph$ in m/s.

$72mph$
$\downarrow \times 1.6$
$115.2km/h$
$\downarrow \times 1000$
$115200m/h$
$\downarrow \div 60$
$1920m/min$
$\downarrow \div 60$
$32m/sec$

sparx

U151 U910

Key Words
Acceleration
Velocity
Speed
Time
Units

1) Use 5 $miles$ = 8 km to write $60mph$ in km/h
2) Write $60km/h$ in m/s
3) Write $6m/s$ in mph

ANSWERS 1) 96 km/h 2) $16.7m/s$ 3) 13.5 mph