INTEGERS, ROUNDING AND PLACE VALUE

Key Concepts

Digits are the individual components of a number.

Integers are whole numbers.

Rounding rules:
A value of 5 to 9 rounds the number up.
A value of 0 to 4 keeps the number the same.

Examples

Order the following numbers starting with the smallest:

1) 5, -3, 4, 7, -2
-3, -2, 4, 5, 7

2) 0.067 0.6 0.56 0.65 0.605
Rewrite 0.067, 0.600, 0.560, 0.650, 0.605
0.067 0.56 0.6 0.605 0.65

Round 3.527 to:

a) 1 decimal place
3 . 5 | 2 7 → 3.5

b) 2 decimal places
3 . 5 2 | 7 → 3.53

c) 1 significant figure
3 | 5 2 7 → 4

sparx
M696
M365

Key Words

Integer
Even
Digit
Odd
Decimal place
Significant figures

A) Order the following numbers starting with the smallest:
1) 6, -2, 0, -5, 3 2) 0.72, 0.7, 0.072, 0.07, 0.702

B) Round the following numbers to the given degree of accuracy
1) 14. 1732 (1 d.p.) 2) 0.0568 (2 d.p.) 3)3418 (1 S.F)

ANSWERS: A1) -5, -2, 0, 3, 6 2) 0.07, 0.072, 0.7, 0.702, 0.72
B1) 14.2 2) 0.06 3) 3000

FRACTIONS, DECIMALS AND PERCENTAGES

Key Concepts

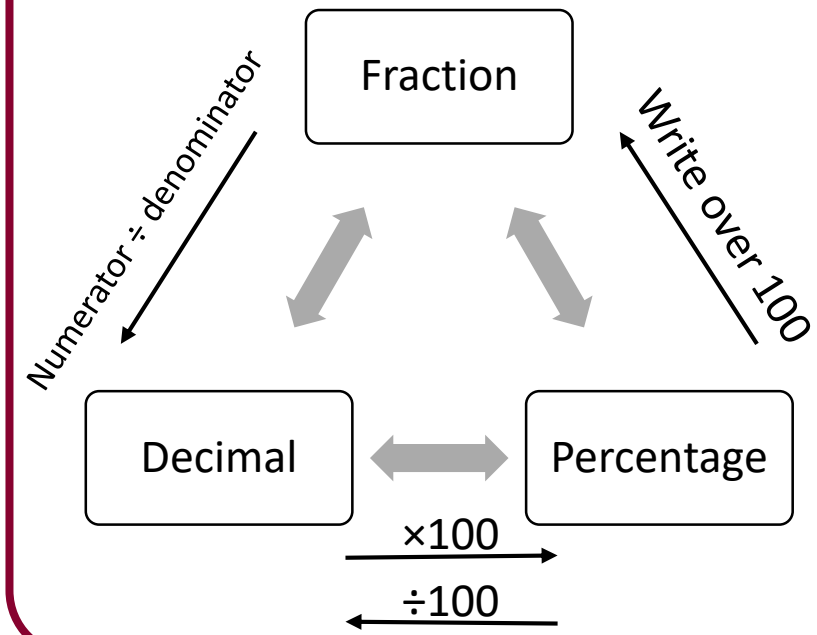
A **fraction** is a numerical quantity that is not a whole number.

A **decimal** is a number written using a system of counting based on the number 10.

Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths
8	7	6	5	.	4	3	2

A **percentage** is an amount out of 100.

Examples



Order the following in ascending order:

$\frac{3}{5}$ 62% 0.67 $\frac{7}{10}$ 0.665

×20 → $\frac{60}{100}$; ×10 → $\frac{70}{100}$; ×100

60% 62% 67% 70% 66.5%

$\frac{3}{5}$ 62% 0.665 0.67 $\frac{7}{10}$

sparx

M958

M264

M922

Key Words

Fraction
Decimal
Percentage
Division
Multiply

1) Convert the following into percentages:
   a) 0.4 b) 0.08 c) $\frac{6}{20}$ d) $\frac{3}{25}$
2) Compare and order the following in ascending order:
   $\frac{3}{4}$ 76% 0.72 $\frac{4}{5}$ 0.706

ANSWERS 1a) 40% b) 8% c) 30% d) 12% 2) 0.706 0.72 $\frac{3}{4}$ 76% $\frac{4}{5}$

FRACTIONS

Key Concepts

$\frac{x}{y}$ → Numerator / Denominator

Equivalent fractions have the same value as one another.

Eg. $\frac{1}{4} = \frac{2}{8} = \frac{3}{12}$

Examples

Calculate $\frac{4}{5}$ of 65:

$65 \div 5 = 13$ — Divide by the denominator

$13 \times 4 = 52$ — Multiply this by the numerator

$\frac{4}{5}$ of a number is 52, what is the original number?

$52 \div 4 = 13$ — Divide by the numerator

$13 \times 5 = 65$ — Multiply this by the denominator

Order these fractions in ascending order:

$\frac{2}{5}$ ×6 → $\frac{12}{30}$ (1)

$\frac{1}{2}$ ×15 → $\frac{15}{30}$ (3)

$\frac{5}{6}$ ×5 → $\frac{25}{30}$ (4)

$\frac{7}{15}$ ×2 → $\frac{14}{30}$ (2)

To be able to compare fractions we must have a **common denominator**

sparx

M601, M835, M931, M157, M197, M110, M265, M671

Key Words

Fraction
Equivalent
Reciprocal
Numerator
Denominator

1) Calculate $\frac{2}{7}$ of 56.

2) $\frac{3}{8}$ of a number is 36, what is the original number?

3) Order the following in ascending order: $\frac{2}{3}$ $\frac{5}{6}$ $\frac{3}{8}$ $\frac{7}{12}$

ANSWERS A 1) 16 2) 96 3) $\frac{3}{8}$ $\frac{7}{12}$ $\frac{2}{3}$ $\frac{5}{6}$

4 OPERATIONS WITH FRACTIONS

Key Concepts

An **improper fraction** is when the numerator is larger than the denominator e.g. $\frac{20}{12}$

Converting from a mixed number into an improper fraction:

$$2\ \frac{3}{5} = \frac{(2 \times 5) + 3}{5} = \frac{13}{5}$$

A **reciprocal** is the value that when multiplied by another gives the answer of 1.

Eg. $\frac{1}{8}$ is the reciprocal of 8.

$\frac{2}{5}$ is the reciprocal of $\frac{5}{2}$

Examples

$1\frac{2}{3} + 2\frac{1}{4}$

$= \frac{5}{3} + \frac{9}{4}$ — Convert into an improper fraction

$= \frac{20}{12} + \frac{27}{12}$ — Find a common denominator

$= \frac{47}{12}$

$= 3\frac{11}{12}$ — Convert back into a mixed number

$2\frac{2}{3} - 1\frac{1}{4}$

$= \frac{8}{3} - \frac{5}{4}$ — Convert into an improper fraction

$= \frac{32}{12} - \frac{15}{12}$ — Find a common denominator

$= \frac{17}{12}$

$= 1\frac{5}{12}$ — Convert back into a mixed number

$1\frac{1}{3} \times 2\frac{3}{4}$

$= \frac{4}{3} \times \frac{11}{4}$ — Convert into an improper fraction

$= \frac{44}{12}$

$= 3\frac{8}{12}$

$2\frac{1}{3} \div 1\frac{3}{5}$

$= \frac{7}{3} \div \frac{8}{5}$ — Find the reciprocal of the second fraction….

$= \frac{7}{3} \times \frac{5}{8}$ — …and multiply

$= \frac{35}{24}$

$= 1\frac{11}{24}$

sparx

M601, M835, M931, M157, M197, M110, M265

Key Words

Fraction
Equivalent
Reciprocal
Numerator
Denominator
Improper/Top heavy
Mixed number

Calculate:

1) $1\frac{2}{3} + 2\frac{3}{4}$

2) $3\frac{3}{4} - 1\frac{1}{3}$

3) $3\frac{1}{5} \times 1\frac{2}{3}$

4) $1\frac{3}{5} \div 2\frac{7}{10}$

What is the reciprocal of:

5) $\frac{2}{3}$

6) 9

7) 0.75

ANSWERS A 1) $4\frac{5}{12}$ 2) $2\frac{5}{12}$ 3) $5\frac{1}{3}$ 4) $\frac{16}{27}$ 5) $\frac{3}{2}$ 6) $\frac{1}{9}$ 7) $\frac{4}{3}$

STANDARD FORM

Key Concepts

We use standard form to write a very large or a very small number in scientific form.

Must be $\times 10$
b is an integer

$$a \times 10^{b}$$

Must be $1 \leq a < 10$

Examples

Write the following in **standard form**:

1) 3000 = 3×10^{3}

2) 4580000 = 4.58×10^{6}

3) 0.0006 = 6×10^{-4}

4) 0.00845 = 8.45×10^{-3}

Calculate the following, write your answer in **standard form**:

1) $(3 \times 10^{3}) \times (5 \times 10^{2})$

$3 \times 5 = 15$
$10^{3} \times 10^{2} = 10^{5}$
15×10^{5}
$= 1.5 \times 10^{6}$

2) $(8 \times 10^{7}) \div (16 \times 10^{3})$

$8 \div 16 = 0.5$
$10^{7} \div 10^{3} = 10^{4}$
0.5×10^{4}
$= 5 \times 10^{3}$

sparx

M719
M678
M757

Key Words
Standard form
Base 10

Links
Science

A) Write the following in standard form:
1) 74 000 2) 1 042 000 3) 0.009 4) 0.000 001 24
B) Work out:
1) $(5 \times 10^{2}) \times (2 \times 10^{5})$ 2) $(4 \times 10^{3}) \times (3 \times 10^{8})$
3) $(8 \times 10^{6}) \div (2 \times 10^{5})$ 4) $(4.8 \times 10^{2}) \div (3 \times 10^{4})$

ANSWERS: A1) 7.4×10^{4} 2) 1.042×10^{6} 3) 9×10^{-3} 4) 1.24×10^{-6}
B1) 1×10^{8} 2) 1.2×10^{12} 3) 4×10 4) 1.6×10^{-2}

FACTORS, MULTIPLES AND PRIMES

Key Concepts

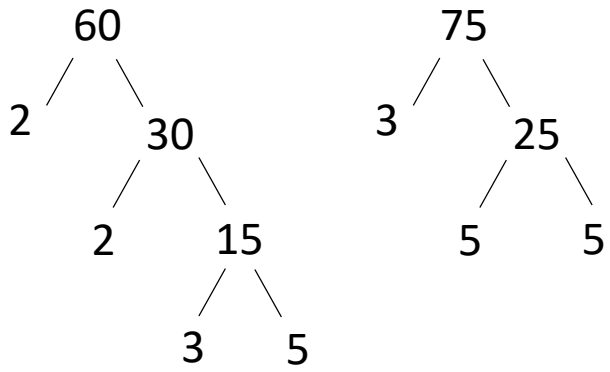
Prime factor decomposition
Breaking down a number into its prime factors

Highest common factor
Finding the largest number which divides into all numbers given

Lowest common multiple
Finding the smallest number which both numbers divide into

Examples

Find the **highest common factor** and **lowest common multiple** of 60 and 75:

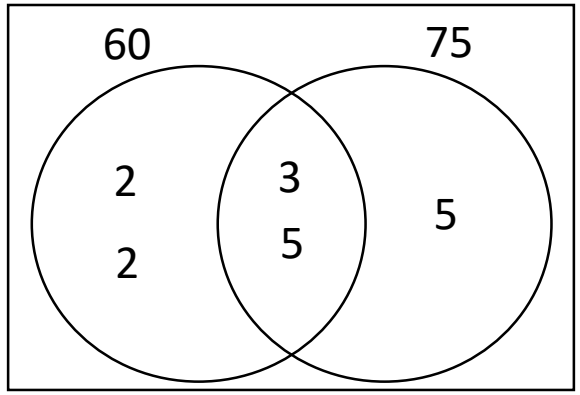


$2 \times 2 \times 3 \times 5$

$2^2 \times 3 \times 5$

$3 \times 5 \times 5$

3×5^2



HCF – Mulitiply all numbers in the intersection

$= 3 \times 5 = 15$

LCM – Multiply all numbers in the Venn diagram

$= 2 \times 2 \times 3 \times 5 \times 5 = 300$

sparx

M108, M698, M365, M227, M365

Key Words

Factor
Multiple
Prime
Highest Common Factor
Lowest Common Multiple

Questions

1) Write 80 as a product of its prime factors
2) Write 48 as a product of its prime factors
3) Find the LCM and HCF of 80 and 48

ANSWERS: 1) $2^4 \times 5$ 2) $2^4 \times 3$ 3) LCM = 240 and HCF = 16

PERCENTAGES

Key Concepts

Calculating percentages of an amount without a calculator:

10% = divide the value by 10
1% = divide the value by 100

Calculating percentages of an amount with a calculator:

Amount × percentage as a decimal

Calculating percentage increase/decrease:

Amount × (1 ± percentage as a decimal)

Calculating a percentage – non calculator:

Calculate 32% of 500g:

10% ⟶ 500 ÷ 10 = 50
30% ⟶ 50 × 3 = 150
1% ⟶ 500 ÷ 100 = 5
2% ⟶ 5 × 2 = 10

32% = 150 + 10 = 160g

Calculating a percentage – calculator:

Calculate 32% of 500g:

$Value \times (percentage \div 100)$
$= 500 \times 0.32$
= 160g

Examples

Percentage change:

A dress is reduced in price by 35% from £80. What is it's **new price**?

$Value \times (1 - percentage\ as\ a\ decimal)$
$= 80 \times (1 - 0.35)$
= £52

A house price appreciates by 8% in a year. It originally costs £120,000, what is the **new value** of the house?

$Value \times (1 + percentage\ as\ a\ decimal)$
$= 120{,}000 \times (1 + 0.08)$
= £129,600

sparx

M433, M905, M476, M533

Key Words

Percent
Increase/decrease
Appreciate
Depreciate
Multiplier
Divide

1) Write the following as a decimal multiplier: a) 45% b) 3% c) 2.7%
2) Calculate 43% of 600 without using a calculator
3) Calculate 72% of 450 using a calculator
4a) Decrease £500 by 6%
b) Increase 65g by 24%
c) Increase 70m by 8.5%

ANSWERS 1a) 0.45 b) 0.03 c) 0.027 2) 258 3) 324 4a) £470 b) 80.6g c) 75.95m

PERCENTAGES AND INTEREST

Key Concepts

Calculating percentages of an amount without a calculator:

10% = divide the value by 10
1% = divide the value by 100

Per annum is often used in monetary questions meaning **per year.**

Depreciation means that the value of something is going down or reducing.

Examples

Simple interest:

Joe invest £400 into a bank account that pays 3% **simple interest** per annum.
Calculate how much money will be in the bank account after 4 years.

3% = £4 × 3
= £12
4 years = £12 × 4
Interest = £48
Total in bank account = £400 + £48
= £448

Compound interest:

Joe invest £400 into a bank account that pays 3% **compound interest** per annum.
Calculate how much money will be in the bank account after 4 years.

$Value \times (1 \pm percentage\ as\ a\ decimal)^{years}$
$= 400 \times (1 + 0.03)^4$
$= 400 \times (1.03)^4$
= £450.20

sparx
M901

Key Words
Percent
Depreciate
Interest
Annum
Simple
Compound
Multiplier

1) Calculate a) 32% of 48 b) 18% of 26
2) Kane invests £350 into a bank account that pays out simple interest of 6%. How much will be in the bank account after 3 years?
3) Jane invests £670 into a bank account that pays out 4% compound interest per annum. How much will be in the bank account after 2 years?

ANSWERS A 1a) 15.36 b) 4.68 2) £413 3) £724.67

COMPOUND INTEREST AND DEPRECIATION

Key Concepts

We use **multipliers** to increase and decrease an amount by a particular percentage.

Percentage increase:

$Value \times (1 + percentage\ as\ a\ decimal)$

Percentage decrease:

$Value \times (1 - percentage\ as\ a\ decimal)$

Appreciation means that the value of something is going up or increasing.

Depreciation means that the value of something is going down or reducing.

Per annum is often used in monetary questions meaning **per year.**

Examples

Compound interest:

Joe invest £400 into a bank account that pays 3% **compound interest** per annum. Calculate how much money will be in the bank account after 4 years.

$Value \times (1 + percentage\ as\ a\ decimal)^{years}$
$= 400 \times (1 + 0.03)^4$
$= 400 \times (1.03)^4$
= £450.20

Compound depreciation:

The original value of a car is £5000. The value of the car **depreciates** at a rate of 7.5% per annum. Calculate the value of the car after 3 years.

$Value \times (1 - percentage\ as\ a\ decimal)^{years}$
$= 5000 \times (1 - 0.075)^3$
$= 5000 \times (0.925)^3$
= £3957.27

sparx

U773, U533, U332, U988

Key Words

Percent
Appreciate
Depreciate
Interest
Annum
Compound
Multiplier

1) Jane invests £670 into a bank account that pays out 4% compound interest per annum. How much will be in the bank account after 2 years?
2) A house has decreased in value by 3% for the past 4 years. If originally it was worth £180,000, how much is it worth now?

ANSWERS A 1) £724.67 2) £159352.71

EXPRESSIONS/EQUATIONS/IDENTITIES AND SUBSTITUTION

Key Concepts

A **formula** involves two or more letters, where one letter equals an **expression** of other letters.

An **expression** is a sentence in algebra that does NOT have an equals sign.

An **identity** is where one side is the equivalent to the other side.

When **substituting** a number into an expression, replace the letter with the given value.

Examples

1) $5(y + 6) \equiv 5y + 30$ is an identity as when the brackets are expanded we get the answer on the right hand side
2) $5m - 7$ is an expression since there is no equals sign
3) $3x - 6 = 12$ is an equation as it can be solved to give a solution
4) $C = \frac{5(F - 32)}{9}$ is a formula (involves more than one letter and includes an equal sign)
5) Find the value of $3x + 2$ when $x = 5$

   $(3 \times 5) + 2 = 17$
6) Where $A = b^2 + c$, find A when b = 2 and c = 3

   $A = 2^2 + 3$

   $A = 4 + 3$

   $A = 7$

sparx

M813, M830, M208, M979

Key Words

Substitute
Equation
Formula
Identity
Expression

Questions

1) **Identify the equation, expression, identity, formula from the list** (a) $v = u + at$ (b) $u^2 - 2as$ (c) $4x(x - 2) = x^2 - 8x$ (d) $5b - 2 = 13$
2) Find the value of $5x - 7$ when $x = 3$
3) Where $A = d^2 + e$, find A when d = 5 and e = 2

ANSWERS: 1) (a) formula (b) expression (c) identity (d) equation 2) 8

REARRANGE AND SOLVE EQUATIONS

Key Concepts

Solving equations:
Working with inverse operations to find the value of a variable.

Rearranging an equation:
Working with inverse operations to isolate a highlighted variable.

In solving and rearranging we **undo the operations** starting from the last one.

For each step in solving an equation we must do the **inverse** operation

Solve:

$5(x - 3) = 20$

Expand

$5x - 15 = 20$

+15 +15

$5x = 35$

÷5 ÷5

$x = 7$

Solve:

$12 = 3x - 18$

+18 +18

$30 = 3x$

÷3 ÷3

$x = 10$

Solve:

$7p - 5 = 3p + 3$

-3p -3p

$4p - 5 = 3$

+5 +5

$4p = 8$

÷ 2 ÷ 2

$p = 2$

Examples

Rearrange to make r the subject of the formulae :

$$Q = \frac{2r - 7}{3}$$

×3

× 3

$$3Q = 2r - 7$$

+7 +7

$$3Q + 7 = 2r$$

÷ 2 ÷ 2

$$\frac{3Q + 7}{2} = r$$

hegartymaths

177-186, 280-284, 287

DELTA Academies Trust

Key Words
Solve
Rearrange
Term
Inverse operation

1) Solve $7(x + 2) = 35$
2) Solve $4x - 12 = 28$
3) Solve $4x - 12 = 2x + 20$

4) Rearrange to make x the subject:

$$y = \frac{3x + 4}{2}$$

ANSWERS: 1) $x = 3$ 2) $x = 10$ 3) $x = 16$ 4) $x = \frac{2y - 4}{3}$

EQUATIONS IN CONTEXT

Key Concepts

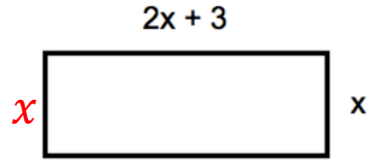
Algebra can be used to support us to find unknowns in a **contextual problem**.

We can always apply a letter to an unknown quantity, to then **set up an equation**.

It will often be used in area and perimeter problems and angle problems in geometry.

Solve to find the value of x when the perimeter is 42cm.

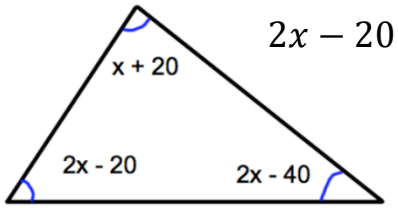
HINT: Write on all of the lengths of the sides.



We know the perimeter is 42cm

$$2x + 3 + 2x + 3 + x + x = 42$$
$$9x + 6 = 42$$
$$6x = 36$$
$$x = 6$$

Angles in a triangle sum to 180°



$$2x - 20 + x + 20 + 2x - 40 = 180$$
$$5x - 40 = 180$$
$$5x = 220$$
$$x = 45$$

Examples

Jane is 4 years older than Tom.
David is twice as old as Jane.
The sum of their ages is 60.
Using algebra, find the age of each person.

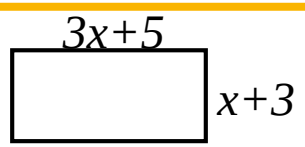
Tom = x → 12
Jane = $x + 4$ → $12 + 4 = 16$
David = $2x + 8$ → $(2 \times 12) + 8 = 32$

$$x + x + 4 + 2x + 8 = 60$$
$$4x + 12 = 60$$
$$4x = 48$$
$$x = 12$$

hegartymaths

DELTA Academies Trust

Key Words
Solve
Term
Inverse operation



1) If the perimeter is 40cm. What is the length of the longest side?

2) Jane is 12 years older than Jack.
Sarah is 3 years younger than Jack.
The sum of their ages is 36.
Using algebra, find the age of each person.

ANSWERS: 1) $x = 3$ therefore the longest length is 14cm 2) Jack = 9, $Jane = 21$, $Sarah = 6$

INEQUALITIES

Key Concepts

Inequalities show the **range** of numbers that satisfy a rule.

$x < 2$ means x is less than 2
$x \leq 2$ means x is less than or equal to 2
$x > 2$ means x is greater than 2
$x \geq 2$ means x is greater than or equal to 2

On a **number line** we use circles to highlight the key values:

○ is used for less/greater than
● is used for less/greater than or equal to

Examples

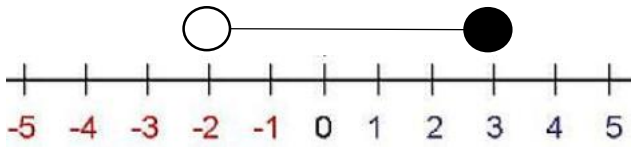
a) State the values of n that satisfy:

$$-2 < n \leq 3$$

Cannot be equal to 2 Can be equal to 3

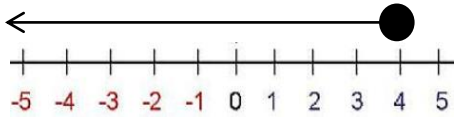
-1, 0, 1, 2, 3

b) Show this inequality on a number line:



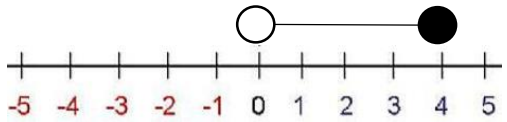
Solve this inequality and represent your answer on a **number line:**

$5x - 6 \leq 14$
+6 +6
$5x \leq 20$
÷5 ÷5
$x \leq 4$



Solve this inequality and represent your answer on a **number line:**

$4 < 3x + 1 \leq 13$
-1 -1
$3 < 3x \leq 12$
÷3 ÷3
$1 < x \leq 4$



sparx
M384
M118
M732

Key Words
Inequality
Greater than
Less than
Represent
Number line

1) State the values of n that satisfy: $-3 \leq n < 2$
2) Solve $4x - 2 \leq 6$ and represent your answer on a number line
3) Solve $5 < 2x + 3 \leq 9$ and represent your answer on a number line

ANSWERS: 1) -3, -2, -1, 0, 1 2) $x \leq 2$ 3) $1 < x \leq 3$