Mathematics

ENLARGEMENT

The Birley Academy

Key Concepts

An **enlargement** changes the size of an image using a scale factor from a given point.
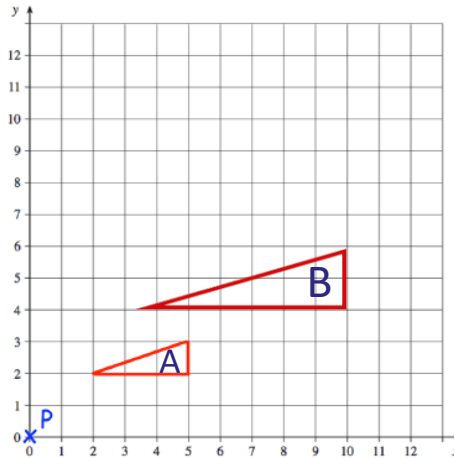
A **positive scale factor** will increase the size of an image.

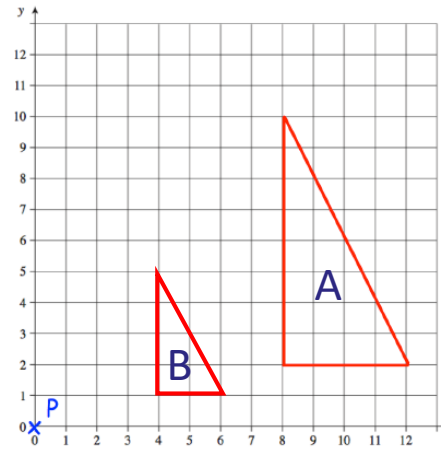A **fractional scale factor** will reduce the size of an image.

A **negative scale factor** will place the image on the opposite side of the centre of enlargement, with the image inverted.
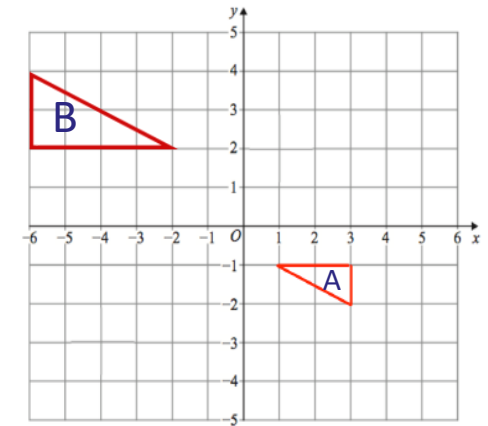
Examples

Enlarge shape A by scale factor 2 from point P.

Enlarge by scale factor $\frac{1}{2}$ from point P.
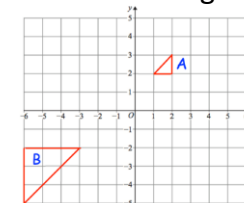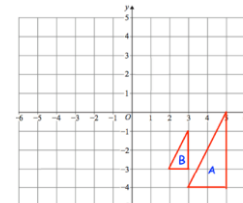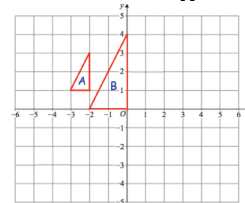

Enlarge by scale factor -2 from (0,0).

sparx

U519
U134

Key Words
Enlargement
Scale factor
Centre
Positive
Negative

Describe the **single** transformation you see on each coordinate grid from A to B:

ANGLE FACTS INCLUDING ON PARALLEL LINES

Key Concepts

Angles in a **triangle equal 180⁰**.

Angles in a **quadrilateral equal 360⁰**.

Vertically opposite angles are equal in size.
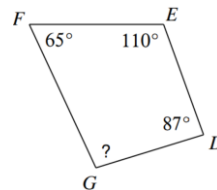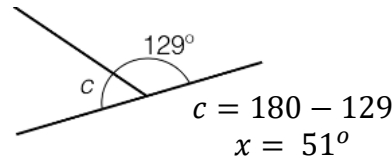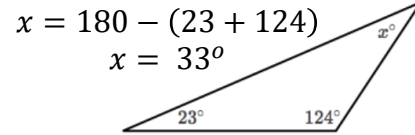
Angles on a **straight line equal 180⁰**.

Base angles in an isosceles triangle are equal.

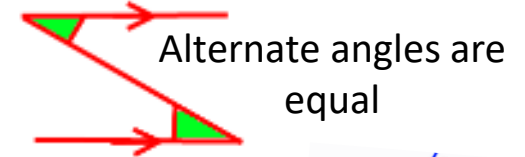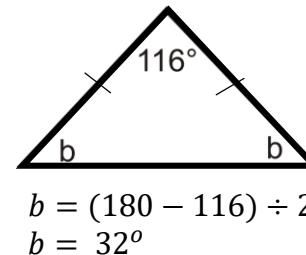**Alternate angles** are equal in size.

Corresponding angles are equal in size.

Allied/co-interior angles are equal 180⁰.
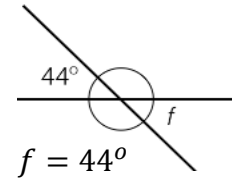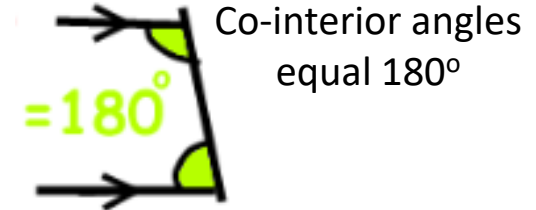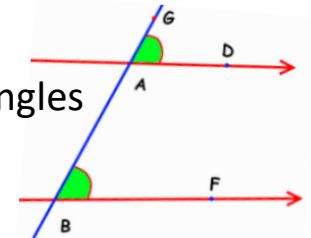
Examples

$x = 180 - (23 + 124)$
$x = 33^o$

$c = 180 - 129$
$x = 51^o$

$? = 360 - (65 + 110 + 87)$
$? = 98^o$

$f = 44^o$

$b = (180 - 116) \div 2$
$b = 32^o$

Alternate angles are equal

Corresponding angles are equal

Co-interior angles equal 180⁰

$=180^o$

sparx

M331
M606

Questions

Calculate the missing angle:

a)

b)
87°
c
94°
b 58°

c)
50° d
e f

ANSWERS: 1) a=50° 2) b=122° c=57° 3) d=130° e=130° f=50°
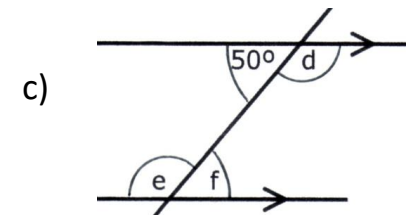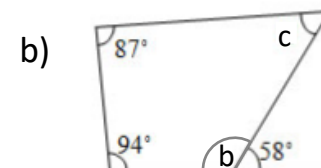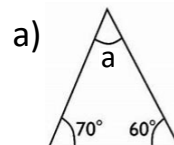
PYTHAGORAS AND TRIGONOMETRY

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Key Concepts

Pythagoras' theorem and basic trigonometry both only work with **right angled triangles**.

Pythagoras' Theorem – used to find a missing length when two sides are known
$$a^2 + b^2 = c^2$$
c is always the hypotenuse (longest side)

Basic trigonometry SOHCAHTOA –
used to find a missing side or an angle

Examples

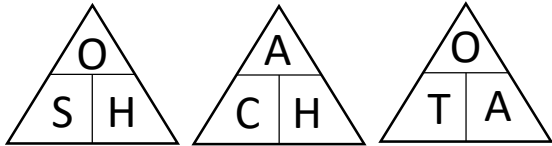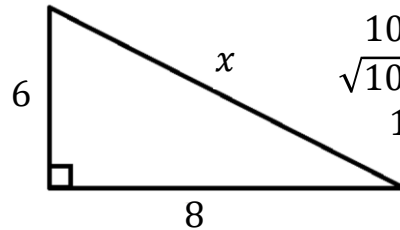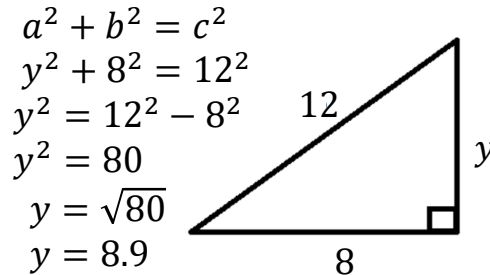### Pythagoras' Theorem

$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = x^2$$
$$100 = x^2$$
$$\sqrt{100} = x$$
$$10 = x$$

$$a^2 + b^2 = c^2$$
$$y^2 + 8^2 = 12^2$$
$$y^2 = 12^2 - 8^2$$
$$y^2 = 80$$
$$y = \sqrt{80}$$
$$y = 8.9$$
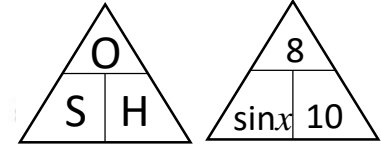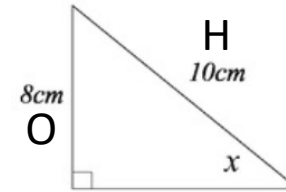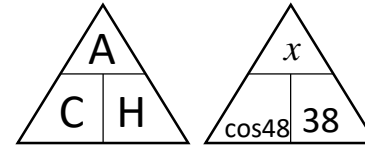
$$\sin x = \frac{8}{10}$$
$$x = \sin^{-1}\left(\frac{8}{10}\right) = 53.1^o$$

$$\cos 48 = \frac{x}{38}$$
$$x = 38 \times \cos 48 = 25.4 m$$
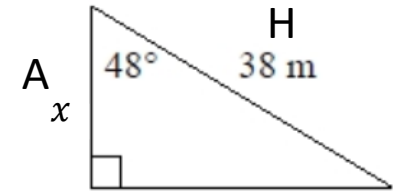
sparx

M677

Questions

Find the value of x.

a)   b)   c) 7cm   d) 4cm

7m   4m   12.5m 11m   4cm   34°
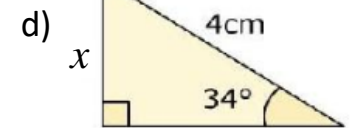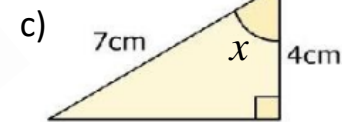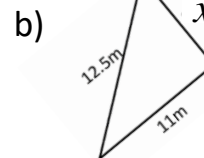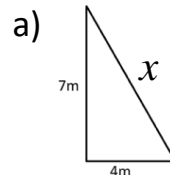
ANSWERS: a) 8.06m   b) 5.94m   c) 55.15°   d) 2.34cm

THE SINE AND COSINE RULE

Key Concepts

Sine rule

To calculate a missing side:

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

To calculate a missing angle:

$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$

Cosine rule

To calculate a missing side:

$$a^2 = b^2 + c^2 - 2bccosA$$

To calculate a missing angle:

$$cosA = \frac{b^2 + c^2 - a^2}{2bc}$$

Area of a triangle using sine

$$area = \frac{1}{2}absinC$$

Examples

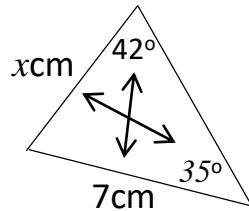$$\frac{x}{sin35} = \frac{7}{sin42}$$

$$x = \frac{sin35 \times 7}{sin42}$$

$$x = 6.0cm$$

$$a^2 = b^2 + c^2 - 2bccosA$$

$$x^2 = 4^2 + 7.5^2 - 2 \times 4 \times 7.5 \times cos42$$

$$x^2 = 27.66$$

$$x = \sqrt{27.66} = 5.26cm$$

$$\frac{sinx}{4} = \frac{sin42}{7}$$

$$sinx = \frac{sin42 \times 4}{7}$$

$$x = sin^{-1}\left(\frac{sin42 \times 4}{7}\right)$$

$$x = 22.5^o$$

$$cosA = \frac{4^2 + 7.5^2 - 8^2}{2 \times 4 \times 7.5}$$

$$A = cos^{-1}\left(\frac{4^2 + 7.5^2 - 8^2}{2 \times 4 \times 7.5}\right)$$

$$A = 82.1^o$$

$$area = \frac{1}{2} \times 4 \times 7.5 \times sin50$$

$$area = 11.49cm^2$$

sparx

U164

Key Words
Sine
Cosine
Side
Angle
Inverse
2D
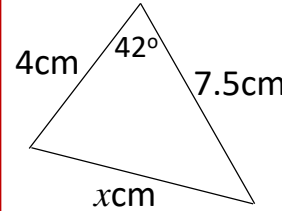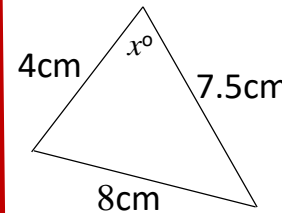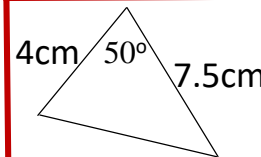
1a) Calculate x
b) Calculate the area of the triangle

2a) Calculate x
b) Calculate the area of the triangle

ANSWERS 1a) 4.57cm b) 9.32cm² 2a) 48.18° b) 20.45cm²

PROBABILITY TREE DIAGRAMS

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Key Concepts

Independent events are events which do not affect one another.

Dependent events affect one another's probabilities. This is also known as **conditional probability**.

Examples

There are red and blue counters in a bag.

The probability that a red counter is chosen is $\frac{2}{9}$.

A counter is chosen and **replaced**, then a second counter is chosen.

Draw a tree diagram and calculate the probability that two counters of the same colour are chosen.

Prob of two reds:
$$\frac{2}{9} \times \frac{2}{9} = \frac{4}{81}$$
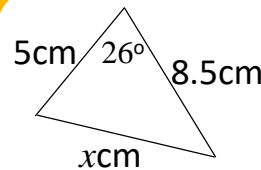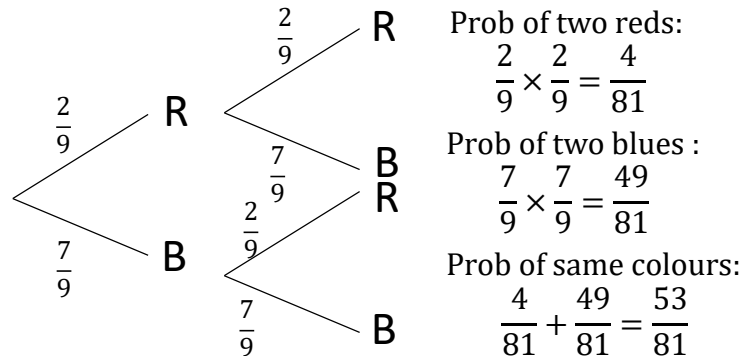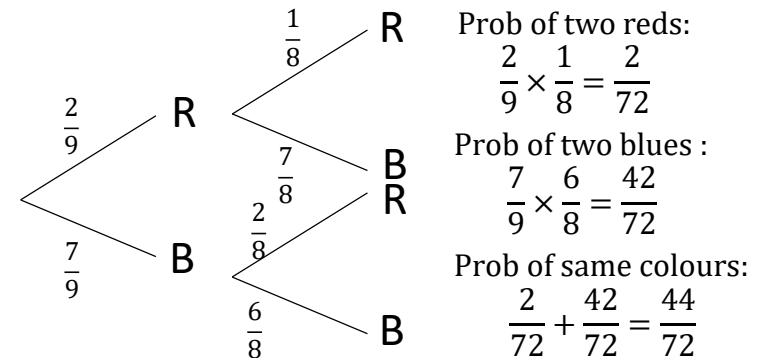
Prob of two blues :
$$\frac{7}{9} \times \frac{7}{9} = \frac{49}{81}$$

Prob of same colours:
$$\frac{4}{81} + \frac{49}{81} = \frac{53}{81}$$

There are red and blue counters in a bag.

The probability that a red counter is chosen is $\frac{2}{9}$.

A counter is chosen and **not replaced**, then a second counter is chosen.

Draw a tree diagram and calculate the probability that two counters of the same colour are chosen.

Prob of two reds:
$$\frac{2}{9} \times \frac{1}{8} = \frac{2}{72}$$

Prob of two blues :
$$\frac{7}{9} \times \frac{6}{8} = \frac{42}{72}$$

Prob of same colours:
$$\frac{2}{72} + \frac{42}{72} = \frac{44}{72}$$

sparx

U558, U729, U821, U806

Key Words
Independent
Dependant
Conditional
Probability
Fraction

1) There are blue and green pens in a drawer. There are 4 blues and 7 greens. A pen is chosen and then **replaced**, then a second pen is chosen. Draw a tree diagram to show this information and calculate the probability that pens of different colours are chosen.

2) There are blue and green pens in a drawer. There are 4 blues and 7 greens. A pen is chosen and **not replaced**, then a second pen is chosen. Draw a tree diagram to show this information and calculate the probability that pens of different colours are chosen.

ANSWERS: 1) 56/121 2) 56/110