UNDERSTANDING PERCENTAGES and FRACTIONS

Key Concept

FDP equivalence

F	D	P
$\frac{1}{100}$	0.01	1%
$\frac{1}{10}$	0.1	10%
$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%

sparx

M429,M152,
M803,M001,M835
M937,M437

Key Words

Fraction: A fraction is made up of a numerator (top) and a denominator (bottom).
Integer: Whole number.
Ascending Order: Place in order, smallest to largest.
Descending Order: Place in order, largest to smallest.

Tip

- A larger denominator does not mean a larger fraction.
- To find equivalent fractions multiply/divide the numerator and denominator by the same number.

Examples

Make the denominators the same.

$\frac{3}{4} \rightarrow \frac{6}{8} \rightarrow \frac{1}{4}$ (4)

$\frac{3}{8} \rightarrow \frac{3}{8} \rightarrow \frac{3}{8}$ (2, 3)

$\frac{1}{2} \rightarrow \frac{4}{8} \rightarrow \frac{1}{2}$ (5, 1)

Convert them all to decimals.

56%	$\frac{3}{4}$	0.871	23%	$\frac{6}{7}$
0.56	0.75	0.871	0.23	0.857…
2	3	5	1	4
23%	56%	$\frac{3}{4}$	$\frac{6}{7}$	0.871

Questions

1) Place these lists in ascending order.

a) $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{12}$ b) $\frac{3}{7}, \frac{1}{2}, 0.49, 0.2$ c) $\frac{7}{32}, 25\%, 0.05, \frac{29}{100}$

ANSWERS: 1) $\frac{7}{12}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$ 2) $0.2, \frac{3}{7}, 0.49, \frac{1}{2}$ 3) $0.05, \frac{7}{32}, 25\%, \frac{29}{100}$

FRACTIONS & PERCENTAGES AS OPERATORS

Key Concept

Multipliers

Find 15%	$\times 0.15$
Increase by 15%	$\times 1.15$
Decrease by 15%	$\times 0.85$

For **reverse percentage** problems you can divide by the multiplier to find the original amount.

Key Words

Percentage: Is a proportion that shows a number as parts per hundred.
Fraction: A fraction is made up of a numerator (top) and a denominator (bottom).
Multiplier: A quantity by which a given number is to be multiplied.

Examples

Non-Calculator

$$\frac{3}{4} \; of \; 32 = 32 \div 4 \times 3 = 24$$

$16\% \; of \; 240$

$10\% = 24$
$5\% = 12$
$1\% = 2.4$

$$= 24 + 12 + 2.4$$
$$= 38.4$$

Calculator

Find **32%** of 54.60 = **0.32** × 54.60 = 17.472

Increase 45 by **12%** = 45 × **1.12** = 50.4

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M157,U475
M958,M264,U88
M437

Tip
There is a % function on your calculator.

To find 25% of 14 on a calculator:
2, 5, SHIFT, (, ×, 1, 4, =

Questions

1) Find these fractions of amounts:
a) $\frac{1}{3} \; of \; 15$ a) $\frac{1}{5} \; of \; 65$ a) $\frac{2}{7} \; of \; 14$ a) $\frac{4}{9} \; of \; 45$
2) a) 35% of 140 b) 21% of 360 c) Increase 60 by 15%

ANSWERS: 1) a) 5 b) 13 c) 4 d) 20 2) a) 49 b) 75.6 c) 69

FRACTIONS, DECIMALS AND PERCENTAGES

Key Concepts

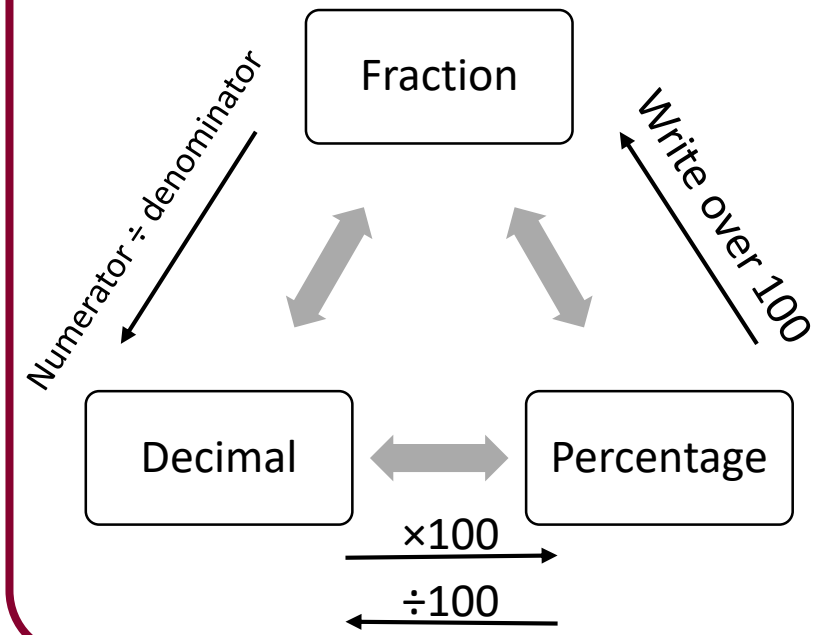
A **fraction** is a numerical quantity that is not a whole number.

A **decimal** is a number written using a system of counting based on the number 10.

Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths
8	7	6	5	.	4	3	2

A **percentage** is an amount out of 100.

Examples



Order the following in ascending order:

$\frac{3}{5}$ 62% 0.67 $\frac{7}{10}$ 0.665

×20 → $\frac{60}{100}$; ×10 → $\frac{70}{100}$; ×100

60% 62% 67% 70% 66.5%

$\frac{3}{5}$ 62% 0.665 0.67 $\frac{7}{10}$

sparx

M958

M264

M922

Key Words

Fraction
Decimal
Percentage
Division
Multiply

1) Convert the following into percentages:
   a) 0.4 b) 0.08 c) $\frac{6}{20}$ d) $\frac{3}{25}$
2) Compare and order the following in ascending order:

$\frac{3}{4}$ 76% 0.72 $\frac{4}{5}$ 0.706

ANSWERS 1a) 40% b) 8% c) 30% d) 12% 2) 0.706 0.72 $\frac{3}{4}$ 76% $\frac{4}{5}$

FRACTIONS

Key Concepts

$\frac{x}{y}$ → Numerator / Denominator

Equivalent fractions have the same value as one another.

Eg. $\frac{1}{4} = \frac{2}{8} = \frac{3}{12}$

Examples

Calculate $\frac{4}{5}$ of 65:

$65 \div 5 = 13$ — Divide by the denominator

$13 \times 4 = 52$ — Multiply this by the numerator

$\frac{4}{5}$ of a number is 52, what is the original number?

$52 \div 4 = 13$ — Divide by the numerator

$13 \times 5 = 65$ — Multiply this by the denominator

Order these fractions in ascending order:

$\frac{2}{5}$ ×6 → $\frac{12}{30}$ (1)

$\frac{1}{2}$ ×15 → $\frac{15}{30}$ (3)

$\frac{5}{6}$ ×5 → $\frac{25}{30}$ (4)

$\frac{7}{15}$ ×2 → $\frac{14}{30}$ (2)

To be able to compare fractions we must have a **common denominator**

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M601, M835, M931, M157, M197, M110, M265, M671

Key Words

Fraction
Equivalent
Reciprocal
Numerator
Denominator

1) Calculate $\frac{2}{7}$ of 56.

2) $\frac{3}{8}$ of a number is 36, what is the original number?

3) Order the following in ascending order: $\frac{2}{3}$ $\frac{5}{6}$ $\frac{3}{8}$ $\frac{7}{12}$

ANSWERS A 1) 16 2) 96 3) $\frac{3}{8}$ $\frac{7}{12}$ $\frac{2}{3}$ $\frac{5}{6}$

PERCENTAGES

Key Concepts

Calculating percentages of an amount without a calculator:

10% = divide the value by 10
1% = divide the value by 100

Calculating percentages of an amount with a calculator:

Amount × percentage as a decimal

Calculating percentage increase/decrease:

Amount × (1 ± percentage as a decimal)

Calculating a percentage – non calculator:

Calculate 32% of 500g:

10% ⟶ 500 ÷ 10 = 50
30% ⟶ 50 × 3 = 150
1% ⟶ 500 ÷ 100 = 5
2% ⟶ 5 × 2 = 10

32% = 150 + 10 = 160g

Calculating a percentage – calculator:

Calculate 32% of 500g:

$Value \times (percentage \div 100)$
$= 500 \times 0.32$
= 160g

Examples

Percentage change:

A dress is reduced in price by 35% from £80. What is it's **new price**?

$Value \times (1 - percentage\ as\ a\ decimal)$
$= 80 \times (1 - 0.35)$
= £52

A house price appreciates by 8% in a year. It originally costs £120,000, what is the **new value** of the house?

$Value \times (1 + percentage\ as\ a\ decimal)$
$= 120{,}000 \times (1 + 0.08)$
= £129,600

sparx

M433, M905, M476, M533

Key Words

Percent
Increase/decrease
Appreciate
Depreciate
Multiplier
Divide

1) Write the following as a decimal multiplier: a) 45% b) 3% c) 2.7%
2) Calculate 43% of 600 without using a calculator
3) Calculate 72% of 450 using a calculator
4a) Decrease £500 by 6%
b) Increase 65g by 24%
c) Increase 70m by 8.5%

ANSWERS 1a) 0.45 b) 0.03 c) 0.027 2) 258 3) 324 4a) £470 b) 80.6g c) 75.95m

PERCENTAGES AND INTEREST

Key Concepts

Calculating percentages of an amount without a calculator:

10% = divide the value by 10
1% = divide the value by 100

Per annum is often used in monetary questions meaning **per year.**

Depreciation means that the value of something is going down or reducing.

Examples

Simple interest:

Joe invest £400 into a bank account that pays 3% **simple interest** per annum.
Calculate how much money will be in the bank account after 4 years.

3% = £4 × 3
= £12
4 years = £12 × 4
Interest = £48
Total in bank account = £400 + £48
= £448

Compound interest:

Joe invest £400 into a bank account that pays 3% **compound interest** per annum.
Calculate how much money will be in the bank account after 4 years.

$Value \times (1 \pm percentage\ as\ a\ decimal)^{years}$
$= 400 \times (1 + 0.03)^4$
$= 400 \times (1.03)^4$
= £450.20

sparx
M901

Key Words
Percent
Depreciate
Interest
Annum
Simple
Compound
Multiplier

1) Calculate a) 32% of 48 b) 18% of 26
2) Kane invests £350 into a bank account that pays out simple interest of 6%. How much will be in the bank account after 3 years?
3) Jane invests £670 into a bank account that pays out 4% compound interest per annum. How much will be in the bank account after 2 years?

ANSWERS A 1a) 15.36 b) 4.68 2) £413 3) £724.67

STANDARD FORM

Key Concepts

We use standard form to write a very large or a very small number in scientific form.

Must be $\times 10$
b is an integer

$$a \times 10^b$$

Must be $1 \leq a < 10$

Examples

Write the following in **standard form**:

1) 3000 = 3×10^3

2) 4580000 = 4.58×10^6

3) 0.0006 = 6×10^{-4}

4) 0.00845 = 8.45×10^{-3}

Calculate the following, write your answer in **standard form**:

1) $(3 \times 10^3) \times (5 \times 10^2)$

$3 \times 5 = 15$
$10^3 \times 10^2 = 10^5$
$\Rightarrow 15 \times 10^5 = 1.5 \times 10^6$

2) $(8 \times 10^7) \div (16 \times 10^3)$

$8 \div 16 = 0.5$
$10^7 \div 10^3 = 10^4$
$\Rightarrow 0.5 \times 10^4 = 5 \times 10^3$

sparx

M719
M678
M757

Key Words
Standard form
Base 10

Links
Science

A) Write the following in standard form:
1) 74 000 2) 1 042 000 3) 0.009 4) 0.000 001 24
B) Work out:
1) $(5 \times 10^2) \times (2 \times 10^5)$ 2) $(4 \times 10^3) \times (3 \times 10^8)$
3) $(8 \times 10^6) \div (2 \times 10^5)$ 4) $(4.8 \times 10^2) \div (3 \times 10^4)$

ANSWERS: A1) 7.4×10^4 2) 1.042×10^6 3) 9×10^{-3} 4) 1.24×10^{-6}
B1) 1×10^8 2) 1.2×10^{12} 3) 4×10 4) 1.6×10^{-2}